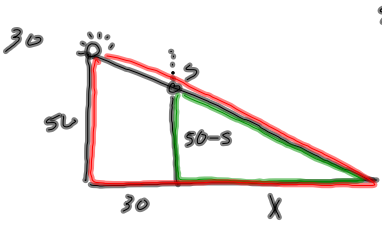


Shadow $\frac{1}{2}$ s later
 $s = 16 + t^2$

Variables: X, s

rates: $\frac{dx}{dt} =$ find
 $\frac{ds}{dt} = 32t = 16$



$$\frac{50}{30+X} = \frac{50-s}{X}$$

$$50X = (30+X)(50-s)$$

$$50\dot{X} = 1500 + 50\dot{X} - 30\dot{s} - X\dot{s}$$

$$30s + Xs = 1500$$

$$30\frac{ds}{dt} + X\frac{ds}{dt} + s\frac{dX}{dt} = 0$$

$$30 \cdot 16 + 345 \cdot 16 + 4\frac{dX}{dt} = 0$$

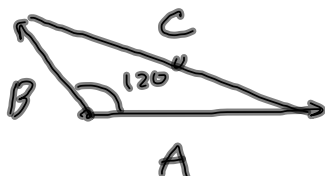
$$\frac{dX}{dt} = \frac{-30 \cdot 16 - 345 \cdot 16}{4}$$

$$\frac{dX}{dt} = -1500 \frac{\text{ft}}{\text{sec}}$$

$s = 16\left(\frac{1}{2}\right)^2 = 4$
 $30 \cdot 4 + X \cdot 4 = 1500$
 $X = \frac{1500 - 120}{4}$
 $X = 345$

Nov 1-7:55 AM

35.

law
of
cosines:

$$A^2 + B^2 - 2AB \cos \theta = C^2$$

$$A^2 + B^2 - 2AB\left(-\frac{1}{2}\right) = C^2$$

$$A^2 + B^2 + AB = C^2$$

$$A: 14 \text{ knot} = \frac{dA}{dt}$$

$$B: 21 \text{ knot} = \frac{dB}{dt}$$

$$DA = 5 = A$$

$$DB = 3 = B$$

$$\text{find } \frac{dC}{dt}$$

Nov 1-8:07 AM

47.

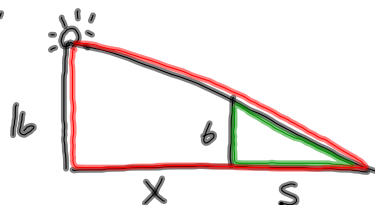
$$y = uv$$

$$\begin{aligned}\frac{dy}{dt} &= u \frac{dv}{dt} + v \frac{du}{dt} \\ &= u \cdot (.05v) + v \cdot (.04u) \\ &= .05uv + .04uv \\ &= .09uv\end{aligned}$$

$$\underline{\text{rel rate}} = \frac{.09uv}{uv} = .09 = 9\%$$

Nov 1-9:43 AM

29.

Variables: x, s

$$\frac{dx}{dt} = -5 \frac{\text{ft}}{\text{sec}}$$

$$\frac{ds}{dt} = \text{find}$$

$$\frac{16}{x+s} = \frac{6}{s}$$

$$16s = 6x + 6s$$

$$10s = 6x$$

$$10 \frac{ds}{dt} = 6 \frac{dx}{dt}$$

$$10 \frac{ds}{dt} = 6(-5)$$

$$\frac{ds}{dt} = \frac{-30}{10} = -3 \frac{\text{ft}}{\text{sec}}$$


Nov 1-8:15 AM

4.6b Related Rates

Pancake batter pours on a griddle at the rate of 1000 cc per minute. The resulting circular pancake is 1 cm thick.

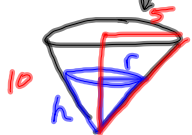
(a) Find an expression for the rate at which the radius grows. What does the expression tell you?

(b) How fast is the radius growing when it is 6 cm?

$\frac{dV}{dt} = 1000$

 Variables: r, V
 rates: $\frac{dr}{dt} = \text{find}$
 $\frac{dV}{dt} = 1000 \frac{\text{cm}^3}{\text{min}}$
 $V = \pi r^2 \cdot 1$
 $\frac{dV}{dt} = \pi \cdot 2r \frac{dr}{dt}$
 $1000 = 2\pi r \frac{dr}{dt}$
 $\frac{dr}{dt} = \frac{1000}{2\pi r}$ as r gets bigger $\frac{dr}{dt}$ gets smaller
 the bigger the pancake the slower it expands
 $\frac{dr}{dt} = \frac{1000}{2\pi \cdot 6} = 26.5258 \frac{\text{cm}}{\text{min}}$

Oct 27-1:51 PM

Water runs into a conical tank at the rate of $9 \text{ ft}^3/\text{min}$. The tank stands point down and has a height of 10 ft and a radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?


 Variables: h, r, V
 rates: $\frac{dh}{dt} = \text{find}$
 $\frac{dr}{dt} = ?$
 $\frac{dV}{dt} = 9$
 $V = \frac{1}{3}\pi r^2 h$
 $\frac{5}{10} = \frac{r}{h}$
 $r = \frac{1}{2}h$
 $V = \frac{\pi}{3} \left(\frac{h}{2}\right)^2 h = \frac{\pi}{3} \frac{h^2}{4} \cdot h = \frac{\pi}{12} h^3$
 $\frac{dV}{dt} = \frac{\pi}{12} h^2 \frac{dh}{dt}$
 $9 = \frac{\pi}{12} \cdot 6^2 \frac{dh}{dt}$
 $9 = 9\pi \frac{dh}{dt}$
 $\frac{dh}{dt} = \frac{9}{9\pi} = \frac{1}{\pi} \frac{\text{ft}}{\text{min}}$

Oct 27-1:53 PM

The volume of a sphere grows at a constant rate of ~~100 cc per min~~ $9 \text{ mm}^3/\text{sec}$

(a) How fast is the radius growing when it is ~~25 cm~~ 3 mm ?

(b) How fast is the surface area growing when the radius is ~~25 cm~~ 3 mm ?



variable: volume = V

radius = r

rates $\frac{dV}{dt} = 9$

$\frac{dr}{dt} = \text{find}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$9 = 4\pi \cdot 3^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{9}{4\pi \cdot 9} = \frac{1}{4\pi}$$

Oct 27-2:03 PM